Tackling Underachievement Through Creativity in the Primary Mathematics Curriculum

October 2013

Publication compiled by Alf Coles & Penny Hay

5x5x5

exploring mathematics through creative practice

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‘centred around the idea of students ‘becoming a mathematician’ and supporting them in working on noticing pattern, making conjectures and finding counter-examples’

Introduction

The aim of this project was to support students who are underachieving in mathematics, through a focus on the creative processes of the subject. What we took creativity to mean, in this context, was:

Students asking their own questions

Students following their own lines of enquiry

Students choosing their own methods of representation

Students noticing patterns

Students making predictions or conjectures

The project was a collaboration between the University of Bristol and the charity “5x5x5=creativity” (5x5x5, see Bancroft et al., 2008). One prompt for this work was the idea of trying out a mathematics based 5x5x5 project, where instead of an artist going into schools, Alf would act as a mathematician and aim to provoke mathematical thinking and creativity. Penny Hay (Director of Research, 5x5x5=creativity) would act as a mentor on the project.

Alf’s own teaching background was in secondary (ages 11-18) education. He had developed some conviction about productive ways of working in the secondary school where he was head of the mathematics department. These ways of working, influenced by Gattegno (1970, 1971, 1974), centred around the idea of students ‘becoming a mathematician’ and supporting them in working on noticing pattern, making conjectures and finding counter-examples (see Coles, 2013). At the start of the 5x5x5 project, we had no idea how relevant or useful these ideas would be in a primary school context (children aged 4 to 11).

We would like to thank the staff and students in the following schools for their unfailing enthusiasm about becoming mathematicians!

Colerne Church of England Primary School, Wiltshire

St Andrew’s Primary School, Bath

St Michael’s Junior School, Twerton, Bath

St Saviour’s Infant School, Bath

Twerton Infant School, Bath

If you would like to know more about the project or are interested in getting involved, please contact:

Alf Coles (alf.coles@bris.ac.uk) or

Penny Hay (p.hay@bathspa.ac.uk).

Outcomes and outputs

The outcomes defined for the project were:

1. Initiating a productive network of teaching staff engaged in their own development.
2. Increasing our understanding of the role of creativity in the primary mathematics curriculum.
3. Supporting previously low attaining students, through creativity, to become successful mathematicians.

As planned, the project involved work with 3 schools in 2011-12 and 5 schools in 2012-13. Alf Coles worked with each of these schools, initially visiting for one observation and then to lead sessions in the teachers’ classes.

In total we have therefore worked intensively with 8 teachers (1 from each school, each year) and approximately 200 primary school children, many of them coming from areas of social deprivation. Headteachers, in evaluating the impact of this project, have written about:

- Raised teacher aspirations; teachers gaining confidence and enjoyment from exploring mathematics; significant interest amongst all members of staff (Outcome 2).
- Raised student attainment; the positive impact of students seeing themselves as ‘mathematicians’; students gaining confidence and enjoyment from exploring mathematics; significant improvement in student attainment (Outcome 3).

‘from a ‘light touch’ leadership level I feel that this style of mathematical approach is empowering the less able children, promoting a sense of fun and enthusiasm in maths’

One headteacher wrote:

‘On average, children enter our school with abilities well below national expectations due to issues of serious social deprivation. This year, for the first time ever, our 7 year old children involved with the project, performed significantly better than children nationally as demonstrated by DfE statistics. The work that you have carried out … has been an inspiration to our school staff and to other schools’

(Letter of support, 25/2/13)

As mentioned in this letter, the impact of the project has extended to other schools, in a manner we did not predict. As a result of dissemination work we planned, we received requests from schools, Universities and Local Authorities, to run sessions with their staff. By November 2013, well over 700 people will have attended events where results from the project were disseminated.

Two other schools evaluated the work of the project in the following terms.

School 1 (comments from Deputy Headteacher):

‘SEN were able to work independently at their own level’

‘Children were able to access mathematics at their own level’

‘They were able to explain what they did and were keen to share that at the end of the lesson’

‘Adults encouraged them to ask more questions that could be explored’

‘from a ‘light touch’ leadership level I feel that this style of mathematical approach is empowering the less able children, promoting a sense of fun and enthusiasm in maths and developing childrens’ desire to talk about their learning. The teachers need PD with regards to questioning to allow and facilitate this style of investigative learning and how to ensure the HA set ‘challenging’ tasks for themselves. In short it should be a school wide focus!’

School 2 (comments from Headteacher):

‘There has been a real impact in 2 key areas for the school’s development’

‘Raising teacher’s aspirations through engagement in well focused and richly documented research. The teachers involved have been able to share the work with their colleagues and have raised their expectations for the possible achievement of the target group of pupils. The creative teaching approaches that have been modelled and explored with the teachers have enriched our school’s maths curriculum provision’

‘Raising pupil’s achievement in maths, particularly for the children who previously lacked confidence in handling number and some mathematical ideas. The open-ended and exploratory nature of the work that you lead helps children to feel secure when approaching new concepts and has helped to increase their mathematical resilience. This was most clearly evidenced in our Year 2 pupils’ end of year SATs results. The opportunity for the pupils to take the lead in a project Inset day was also a real high point for them’

What we did and what we learnt

Teachers on the project met as a group six times a year, to reflect on activities and plan for the coming term. Following the initial meeting in each year, Alf began weekly visits to each school, focused particularly in the Autumn and Spring terms. There would always be one visit for Alf to meet the class and observe them working with their teacher. After that, Alf generally led sessions, having discussed with the teacher the focus. Activities were usually planned to last more than one session and in some schools lasted for four sessions over a period of more than 1 month. Many teachers expressed surprise at the students’ ability to re-engage with activities after a break of a week and to sustain their interest in an activity over a month or more.

In the summer of 2012, we organised a mathematics conference led by the students themselves (we think this may have been a global first!). Students planned sessions for each other, based on the mathematics they had done over the year and presented activities to each other. The feedback from the day was highly positive – students loved being in the position of teacher and sharing their expertise. In the summer of 2013, we took a virtual approach and schools video recorded challenges to each other, which have been shared in a DropBox folder. We are hoping schools will respond with video responses and further new challenges.

‘A striking commonality has been the enthusiasm students have had for mathematics in the project lessons. We tracked student progress and in many cases there were remarkable achievements’

A striking commonality has been the enthusiasm students have had for mathematics in the project lessons. We tracked student progress and in many cases there were remarkable achievements, for example, children on School Action making 4 sub-levels of progress in a year. What the statistics cannot show, however, is the potential long-term benefits from students bringing positive attitudes to their study of mathematics.

We have learnt many things over the last 3 years and the following ideas seem to us to have been central to the success of project classrooms.

A focus on structure

Students can work with big numbers from the earliest years of school, if they are presented in a structured manner (see Case Study 3). Working with big numbers has invariably generated interest and enthusiasm. Students can gain a sense of the power of mathematics. For example, if you know that 3 follows 2, there is little else you need to know to be able to say the number after 172. Extending early number work to hundreds and thousands has exposed students to the regularity and structure of the number system. This regularity and structure is hidden when work is limited to the numbers 1 to 20.

A focus on pattern

In all schools, we offered students the purpose for their work of ‘becoming a mathematician’ and the associated mathematical processes of looking for pattern and making predictions were emphasised at every opportunity.

The focus on pattern supported students in extending their work into big numbers and again was central to developing positive attitudes towards the subject.

A focus on talk

In all classrooms, students were encouraged to discuss their work with partners and, for example at the end of a lesson, with the class. From students’ own work, others would be invited to raise questions that could form the basis for on-going activity. Students seeing each other noticing patterns proved in many cases to be a good way to get the practice to spread.

Mixed attainment groupings

One implication of wanting to get students talking about their mathematics has been the desirability of finding starting points for activities that all students can access. In turn, having the same mathematical task (that is then differentiated by resource and by outcome) has freed teachers to experiment with pairings and groupings not defined by prior-attainment. Teachers across schools have commented on the positive benefits of mixed groupings, for example in allowing students who have been under-attaining to begin to view themselves differently in relation to mathematics.

Teachers ‘getting alongside’ their students

The notion of ‘becoming a mathematician’ freed many teachers to unhook from feeling they needed to know the answer to every question. When a student asked a question, teachers would often reflect it back and ask, as mathematicians, how we might go about finding the answer. Ultimately, many of the project teachers reflected that the changes in their classrooms were as much about their manner of interacting with students as with finding different activities.

Using teaching skills from other subject areas

Another common finding was teachers describing how they began to introduce teaching strategies they were comfortable with in other subjects, to their work in mathematics. For example, one teacher used a ‘talk chair’ in combined humanities lessons, and introduced this into mathematics lessons as well. Another teacher recognised that in other lessons, she was encouraging of students to choose their own methods of representing their work, but had never done this in mathematics before.

Listening and documenting

The creative processes within the project, for teachers and educators, have been important. The charity 5x5x5 places emphasis on the importance of documenting learning and all the teachers on the project have commented on the significance of having the opportunity to watch their classes and perhaps see students behaving differently. There was symmetry in the processes offered to teachers and students in terms of supporting an exploration of questions that are meaningful to them and encouraging documentation, with choice about how this is done.

Case Studies

Case Study 1: Henry's Journey

The next sections of this booklet are three case studies, one of a student, one a teacher and one of a resource. These studies are included to give a more in-depth sense of the impact of the project in relation to the intended Outcomes.

Case Study 1 (Henry's Journey) gives a sense of how one student has transformed his relationship to the study of mathematics (Outcome 3).

Case Study 2 (Hannah's Journey) indicates how we have been successful in leading staff development to a far wider extent than anticipated (Outcome 1).

Case Study 3 (The Gattegno Chart) in part, contains reflections on why we think we have been successful and how our understanding of the role of creativity in learning mathematics has developed (Outcome 2).

Looking forward, one area we would like to research further is how our approach may offer under-attaining students a crucial bridge to an awareness of place value. Two-digit place value has been seen as a key component of success at the end of year 2, and success at that age is correlated strongly with success at age 16. Case study 1 details the development of a student who, at the start of year 2, was under-attaining and destined to be in the group of students likely to fall below a 'C' at GCSE. By the end of the year, he met government expectations for his year group, meaning he is now highly likely to achieve well in mathematics.

Henry's Journey

This case study follows the work of one child and charts his development, particularly in terms of being able to work with pattern in mathematics. The student was chosen for this study because his teacher had decided to document his development (i.e., not because he was necessarily different or remarkable in terms of the progress he made compared to others in the project). Figure 1 shows Henry's (a pseudonym) work in November 2011 and Figure 2 his work in May 2012.

It is evident he has moved from some uncertainty about the processes or symbols of addition and subtraction to being able, by the end of the year, to solve a relatively complex examination question. While our focus has been on supporting student creativity, we do want students to succeed in formal assessments as well and there is evidence in this case study that there is no contradiction between these agenda. Through getting excited about 'being a mathematician' and noticing pattern and working with big numbers, Henry has developed his awareness of number to the point where the exam question in Figure 2 was unproblematic.

We believe a key factor in Henry's striking development came through his appreciation of the role of pattern in mathematics, coupled with his teacher's growing emphasis on the processes of mathematical thinking (including providing repeated opportunities to spot and use pattern).

Figure 1: Harry in Nov 2011

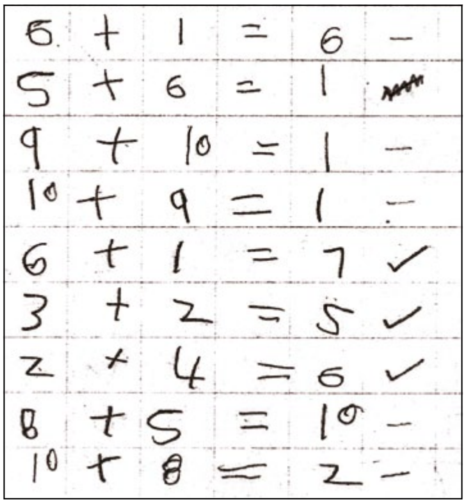
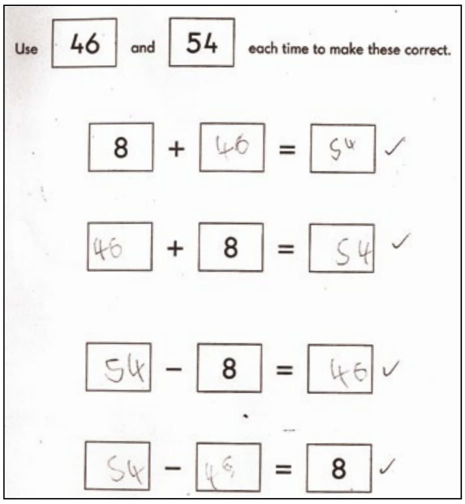


Figure 2: Harry in May 2012



‘Looking forward, one area we would like to research further is how our approach may offer under-attaining students a crucial bridge to an awareness of place value’

In January 2011 (Figure 3) Henry can be seen to have extended a pattern in the five times table, up to 14 multiplied by 5. His teacher recorded two comments Henry made: “It’s all five or zero” and in response to a question about how he had done this work: “I used a hundred square and used the pattern”.

By March 2012 (Figure 4) Henry demonstrated that he was able to complete a ‘journey’ of multiplication and division by 10 (see Case Study 3 for some of the context of this activity), apparently working confidently with numbers as big as 90,000.

In April 2012, the emphasis on pattern in Henry’s written work continues. In the work in Figure 5, rather than needing a teacher to write down his comments (as in January 2012, Figure 3), he is able to write for himself about the pattern he noticed: “Inside goes up in order, outside goes up in twos”.

Henry has been able to work in a systematic manner (increasing the size of his rectangle by 1 unit each time) and, as a result, has been able to notice a pattern in his results. Henry’s teacher wrote (in her own learning journal):

> *‘Henry continued this (see Figure 5) over 2 days, keen to carry on and use patterns he had spotted.’*
>
> *‘He used I for inside and O for outside.’*
>
> *‘We used the word area when talking about it together.’*
>
> *‘Henry is discovering links between area and perimeter.’*

In May 2012, Henry was working on the link between multiplication and addition. He wrote some multiplication sentences, 4x2=8 and then wrote out 2+2+2+2=8, using images of beads to help him.

His teacher wrote:

> *‘Discovery focus for Henry today was to investigate how + and x are linked. Henry sensibly took the 2s to work with because he felt confident at counting in 2s. After making some mistakes, he noticed the link and could explain it. Further, he could take the pattern into answering questions about higher multiplication facts.’*

Figure 3: January 2012

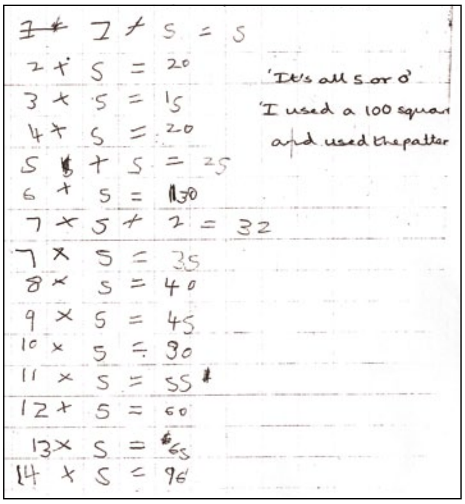


Figure 4: March 2012

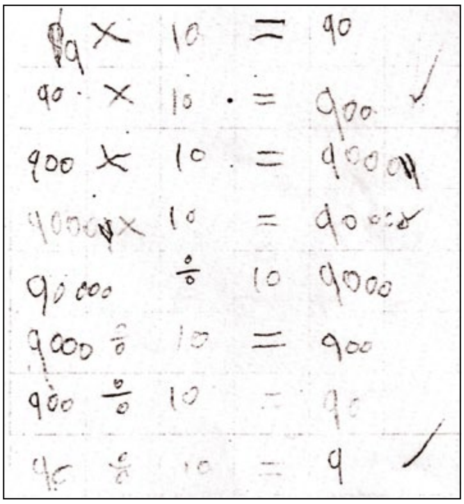
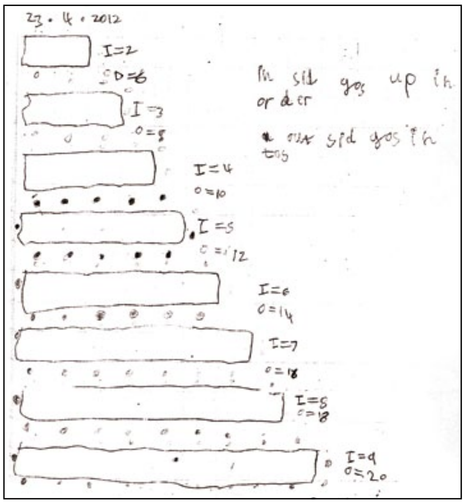
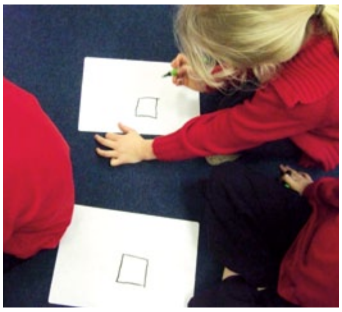


Figure 5: April 2012





'I like being a mathematician because it's fun when you can just keep on going. Numeracy makes me happy because I just want to do more numbers'

There is a shift here into noticing a pattern (April) and then into noticing a pattern and being able to extend it (May).

Henry was asked to reflect on his own mathematical learning journey, in the summer of 2012. We end the case study with his words, that seem linked powerfully to Outcome 3 and students developing confidence in mathematics:

> *'I love doing times tables on my journeys. And I love doing Cuisenaire because it helps me do the right writing. If I want to write the sum, I can just use the right colour.'*
>
> *'Numbers are easier if I spot patterns. If it goes 10, 20, 30 it will be a pattern. When Alf came I used big numbers and I was happy. If I use small numbers at the start it will get bigger and bigger. If you start with small numbers it helps you do big numbers.'*
>
> *'I like being a mathematician because it's fun when you can just keep on going. Numeracy makes me happy because I just want to do more numbers.'*

Case Study 2: Hannah's Journey

Hannah's Journey

Hannah taught a year 2 group in 2011-12 when she was one of the teachers on the project. Her school serves a catchment area with high levels of deprivation. In Alf's first session with her class, Hannah had wanted him to work on problem-solving in the context of money. Towards the end of the session, Alf got all the children to sit down near the front of the classroom and talk about what they had done and found out. A girl, J, put her hand up and Alf asked her to speak. She proceeded to talk to the whole group for several minutes. It appeared that what she was doing, in part, was reading a long list of numbers from her book. After the lesson, Hannah and Alf discussed this incident. Neither Alf nor Hannah had made much sense of what she had been saying. Hannah at the time had been concerned about allowing her so much space to talk. She was a student who was underachieving in mathematics. Hannah had been worried about what the rest of the class were doing during these minutes when J was talking in a pretty unintelligible manner. Alf had chosen to give her this time and felt it had been important to do this, whilst recognising Hannah's concerns.

At a teacher meeting on 21st March 2012, Hannah was reflecting on the progress her students had made in terms of being able to spot patterns that were allowing them to make progress in mathematics. She spoke of her students loving mathematics now and being excited to show other staff the patterns they had noticed and the work they had been able to do. She then said, without any prompting:

> *'I think it goes back to that very first session we did when um you let J read those numbers because at that very beginning its her trying to spot something and other children are spotting (.)*
>
> *and to us it didn't really make any sense (.)*
>
> *and it's like letting children (.)*
>
> *like M for example going 'I used a pattern I did two two two two two' cos he's added two every time (.)*
>
> *and just allowing them to say that out and then gradually you see actually through this that they've then actually begun to spot patterns that they can use and that are helpful (.)*
>
> *so yeah going back to that first very beginning when you said you thought it was really important to let her do it'*
>
> (Hannah, Teacher meeting, 21-3-12)

There is a shift here in Hannah's thinking, from feeling that allowing J space to talk was not worth the waste of other students' time, to seeing it as being a crucial moment in students coming to value their own voice in the context of doing mathematics.

‘We have worked with numbers I wouldn’t have got to with year 2 before but patterns led us there. Some children are at the beginning of algebra by using letters as well as numbers to record patterns’

Hannah kept her own learning journal, documenting her thoughts during the year and the following quotations are taking from this. Thinking back to November 2011, Hannah wrote:

> *‘I was also continuing to amend my own daily maths lessons through the way I planned. My plenaries became time for ‘talk chair’, which gave children the opportunity to talk us through their discovery and reflect on what they had noticed and where they wanted to go next. This time is very beneficial for everyone as the children would respond to what they saw and heard by adding in theirs or suddenly spotting the pattern for themselves.’*
>
> *‘My carpet teaching time transitioned into a time for lots of talk about a given statement, sequence, objects in the middle of a circle. Children began to demonstrate their talk through backing up what they said by writing a number sequence or pattern to go with it.’*

In relation to her teaching in the Spring of 2012:

> *‘Recently, I put up the Fibonacci sequence and in less than 2 minutes, 2 separate groups of children could see and explain the pattern!*
>
> *We have worked with numbers I wouldn’t have got to with year 2 before but patterns led us there. Some children are at the beginning of algebra by using letters as well as numbers to record patterns. Their understanding of place value has been reinforced and used regularly through working with 10s, 100s, 1000s and even for some, tens of thousands and hundred thousands!’*

Reflecting back over the year:

> *‘In my class, I have a large group of children who find anything too set and structured difficult to engage with and learn from but as soon as you give them a bit of freedom, they show you what they are made from! One boy in my class has finally found his love of maths and ability to do it! I realised that when he had the same set activity as his differentiated group, he often rejected it because he couldn’t cope with the feeling of having to ‘compete’ with the others on his table. Now that he is able to choose the numbers he works with, even if the investigation is the same as his table, to him he is doing his own thing and therefore feels he has control and confidence in his learning.*
>
> *Encouraging child-led learning in maths (spotting patterns, posing questions, making conjectures) has developed my class into well rounded mathematicians, who love maths!’*

In November 2012, Hannah spoke to an audience of 60 about her learning over the year and the approach to teaching she had developed. This event was part of the Economic and Social Research Council’s ‘Festival of Ideas’ at the University of Bristol. Resulting directly from this session, Hannah was invited to lead staff development sessions at: Southdown Junior School (Bath), The Mead Community Primary School (Trowbridge), Newbridge Primary School (Bath), Redcliffe Children’s Centre (Bristol) as well as leading a session for students on the Post Graduate Certificate of Education course at Bath Spa University.

It is evident that the story of one teacher’s journey can be a powerful catalyst for change in others. The interest in the local area, and more widely, in the outcomes of the project has led to a loose network of interested staff (Outcome 1), far wider than we had originally anticipated.

‘The (formal) symbols in the chart are not linked to anything concrete, in the sense of collections of objects - the chart and the work in chanting offer connections between the symbols themselves’

Case Study 3: The ‘Gattegno’ Tens Chart

The ‘Gattegno’ Tens Chart

The tool that teachers on the project spoke about as providing the most innovation in their teaching has been the Gattegno tens chart (Figure 6).

Alf used the chart at some point in each of the schools to set up a range of activities and teachers continued to use the chart and develop ways of working on their own.

We exemplify the use made of the Gattegno chart through a narrative account of one project in one school, which lasted four 1-hour sessions, each of them one week apart. This project was chosen for analysis as it was the most fully documented one, which made use of the Gattegno chart. Alf had been asked by the teacher to support the students in working on multiplication and division. The school is a rural state primary school, with levels of attainment in line with UK national averages. Narrative accounts (written in Alf’s voice), taken from field notes and emailed reflections (written soon after the lesson and sent to the group of teachers), are italicised and analysed immediately afterwards, in relation to evidence of symbol use. The class was a mixed year 3-4 (ages 7 to 9) with students from across the range of attainment within the school.

Beginnings

I stuck the Gattegno chart to the wall at the front of the classroom (without the decimal rows, but going up to hundred thousands) and students gathered on the carpet. I began the class by getting the students chanting, to get familiar with how it works. I tapped on a number in the units row and got the class to chant back in unison the number name, then continued for other numbers of the units row and extended to numbers in the tens row. I was conscious of wanting to generate responses from every student, not shouted, but said confidently.

I then wanted to link unit names with tens names. I tap on “4” (class chant FOUR) and then “40” (class chant FOUR-TY); tap on “6” and then “60”; tap on “8” and then “80”. I focus attention on how the number name changes. All we do is add “-ty” to the end of the digit number. We practice this. Having established these names, I tap on “40” followed by “2” – students need to chant back “FOUR-TY-TWO”, again, more practice at saying these two digit numbers.

I tap on a number, and invite the class to chant back the number that is 10 times bigger. The first response needed confirmation and repetition to ensure the whole class called back the answer. I initially chose single digit numbers, and the class chanted back the number 10 times bigger, then progressed to other numbers, returning to single digit ones if the class lost confidence. After a few minutes, I invited someone to say how, on the chart, they were getting their answers. I focused attention on how you can get the answer simply by moving down one row on the chart. Returning to chanting with the awareness of movement, I then pointed on a number and invited the class to chant back that number divided by 10 (which is a movement up one row). I repeated the process for multiplication and division by 100.

Figure 6: An example of a Gattegno tens chart

0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900
1000	2000	3000	4000	5000	6000	7000	8000	9000

The Gattegno chart offers a perceptual structuring of the number system. A linguistic connection between units and tens rows (adding "-ty") is established. The use of chanting and the association of multiplication and division by powers of 10, with visual movements up and down a row aim to establish an immediate (habitual) response to relationships within the chart. The students have to respond to the spoken stimulus 'multiply by 10' and a tap on the chart, by calling out the number in the row below, or 'divide by 10' and the number in the row above. The (formal) symbols in the chart are not linked to anything concrete, in the sense of collections of objects – the chart and the work in chanting offer connections between the symbols themselves. Students are developing habitual responses in relation to the structure and relationships embedded in the chart. At the point I stopped the class and asked what you have to do in order to multiply by 10, (getting answers such as "move up one row") there was an articulation in words of one aspect of this visual structure.

First journeys

I offered a challenge, to choose a number on the chart, go on a journey multiplying or dividing by 10, 100 and to get back to where they started. We did two journeys all together so I could demonstrate how I wanted them written out. Students suggested the movements and I supported the writing. In keeping with the principles of the project, I then asked students what questions they might want to explore linked to their journeys. Students made these suggestions: how big can you make the number? how long can you make the journey? how many ways can you do the same journey? Before students returned to their desks, I checked that everyone knew the number they were going to start with.

The student from Figure 7 completed three journeys and ticked their own work since they recognized that they got back to where they started (we suggested they tick their work in this way). The use of ÷, × suggests a symbolic representation of the movements up and down the chart. There is evidence of students' confident use of inverse operations and the distinction between directions of movement on the chart, linked to the symbols for ÷ and ×. All students were able to produce these kinds of journey, some stayed with the same type of journey done from different starting points, others branched out to try more and different combinations of operations.

In the first lesson, I was struck by one student (see Figure 8) who had chosen a number then done x10, x10, x10, x10 and had worked out that to get back in one go to where she started, she needed to divide by 10,000. I asked her how she knew this and she said to undo x10 you divide by 10, then x10x10 needs divide by 100, x10x10x10 needs you to divide by 1000 and so on. It was interesting that several students wanted to undo x10 x10 by dividing by 20 (rather than 100). This seems like a common misconception and a good one to be working on.

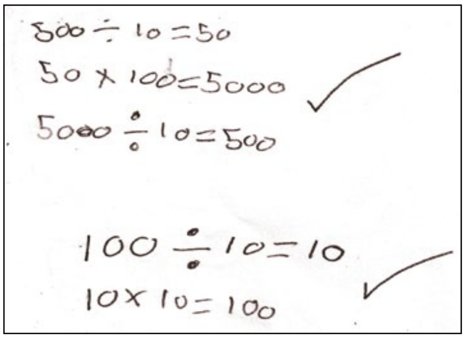
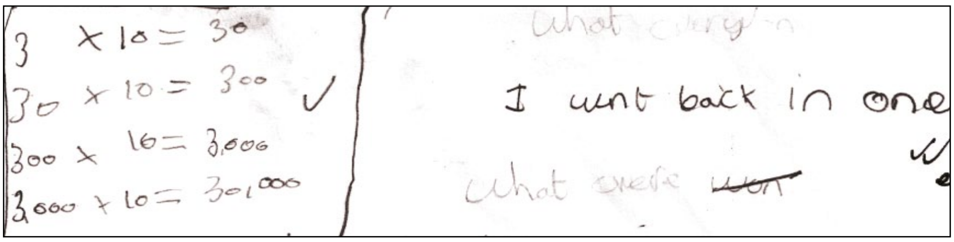


Figure 7: Three 'journeys'

There is evidence here of the student extending the pattern of how to divide by 10 and 100, to work out what she must do as the inverse of x10, x10, x10, x10. Here perhaps we see evidence of a playful exploration of symbols, extending a relationship observed between successive multiplications by 10. As a class we had not worked on division by numbers greater than 100.

We interpret this student as having noticed something about the symbols themselves. This student appears to be getting a 'sense' of how these multiplications and divisions operate and working on relationships between the relationships visible in the chart.

Figure 8: A student extends to division by 10,000: "I went back in one"



Student noticing

Towards the end of the first lesson, I asked all the students to stop their work on the journeys and then spend a couple of minutes writing down anything they had noticed about what they had done, or anything they had felt or any questions they still wanted to ask. We then came to the carpet at the front of the room and students shared some of their ideas with each other.

We interpret the student text as (Fig 9a): "I've noticed it is clever because if by 10 you x 10 then get back" and (Fig 9b): "I found out that I timesed twice but I divided once by timesing by 10 twice and dividing by 100 once".

The student in Figure 9a is articulating a key mathematical awareness, and offers an example of the connection between multiplication and division. This is a representation, expressed in language, of an inverse relationship. A further awareness is offered by a different student in this first session (Fig. 9b).

This student has noticed the inverse connection between multiplication by 10 twice and division by 100. Students are articulating the awarenesses they have developed about the relationships between the symbols they have been using. There is no evidence of these students having made, or needing to make, links between their symbol use and an awareness of the relative sizes of the numbers they are operating with. The symbols for the operators stand for movements or relationships within the chart and, starting from these symbols, students were able to uncover further connections.

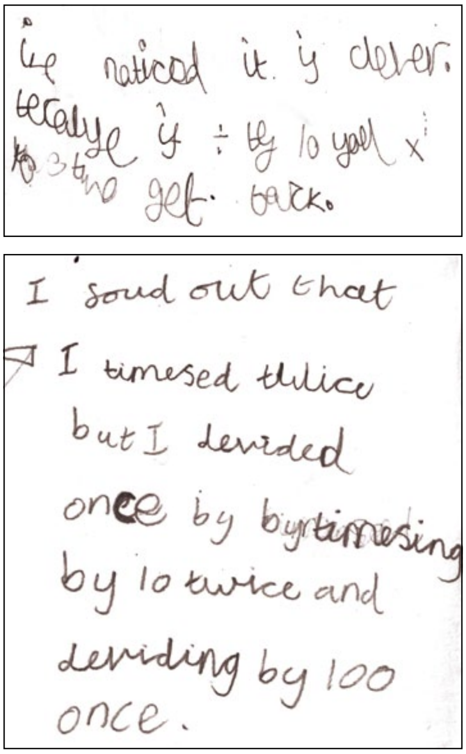
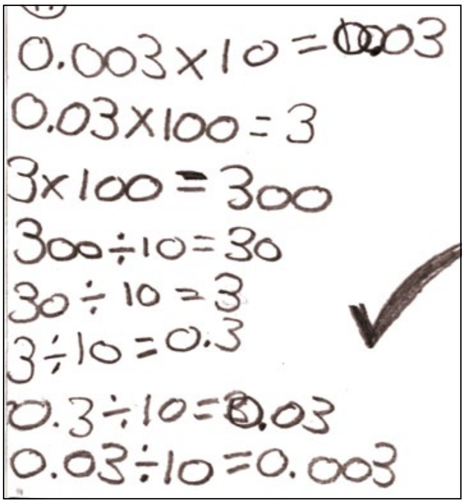


Figure 9a & 9b: Connections made

Figure 10: A journey into decimals



Moving to decimals

A few students had seen decimals rows (printed on the back of the chart I was using) in the first lesson and others had talked about wanting to use them at the end of that class. At the start of the second session (one week later), I worked again with students, chanting multiplication and division by powers of ten, this time moving into decimals as well. Students were invited to continue work on their journeys and again I asked students what questions they wanted to work on, which were written on a flip chart. The idea of 'getting back in one' was shared with the whole class. The structure of the tens chart meant that there was little difficulty for students in extending what they had done with journeys in whole numbers, to journeys into decimals (see Figure 10).

The student who wrote Figure 10 has used the operations ×10, ×100 and ÷10. The combination of these different operations suggests the student has some control of these symbols and is gaining fluency in their manipulation. There is no apparent discontinuity or difficulty in incorporating symbols for decimal numbers into the movements on the chart and the operations with powers of 10. The focus on multiplication and division as a relationship that is expressed visually on the chart means there is no difficulty in extending the operations into decimals.

‘Part of the evidence for the fluency or control over these symbols is the efficiency of the method employed. The student has, in effect, discovered or at least used a unitary method’

Moving away from powers of 10

At the end of the second lesson, one student asked if they could make a journey from one column to a different column (all journeys to this point had been in a single column of the chart, representing multiplication and division by powers of 10). As teachers and researchers we had not expected this question and I had never used the chart to do this. However, we took on the challenge and in the third lesson, I offered the idea that to go from, say, 7 to 4 on this chart, since we only use multiplication and division, we would have to divide by 7 to get to 1 and then multiply by 4 to get to 4. With this new possibility, students were then challenged to choose a starting point and a different end point for their journeys.

An example of the simplest of these new kinds of journey is Figure 11a. A more complex challenge a student set themselves can be seen in Figure 11b. Although there is a small error in the last line (1.01 was written instead of 0.01), this student demonstrates a striking fluency in the use of these symbols. The student who wrote Figure 11b was seen by the school as having low levels of prior attainment in mathematics. Part of the evidence for the fluency or control over these symbols is the efficiency of the method employed. The student has, in effect, discovered or at least used a unitary method. In the most efficient manner possible (in the context of the work on this problem) the student has reduced 3 000 to 1 and then gone from 1 to 0.04, again in the most efficient manner possible (given that movement left and right on the chart has only been offered as a possibility along one row).

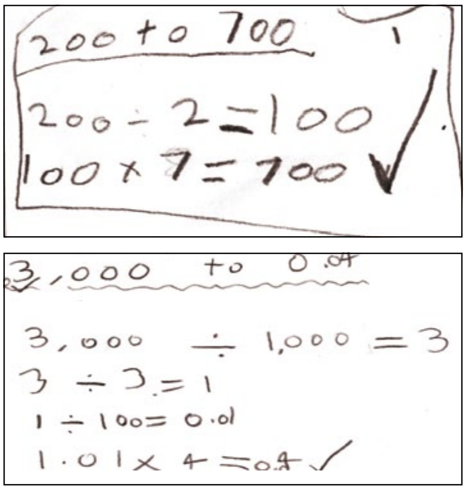


Figure 11a and 11b: Journeys from one column to a different column

Discussion

One of the teachers, in reflecting on this particular sequence of lessons, commented on how she had always, previously, approached multiplication through a concrete manipulation and grouping of objects. In this treatment, we have observed students being able to begin with a visual tool that is already removed from concrete objects (Figure 6) and build further abstractions, through the focus on multiplication and division as a relationship that can be observed within the chart. At some point, no doubt, students will need to re-connect their use of multiplication and division with physical objects, but in this treatment it appears students are able to work with numbers (e.g. decimals) and operations (e.g. division by 10,000) that would not normally be introduced until later in the curriculum in the UK. The work on journeys supported students in gaining awareness of the inverse relation between multiplication and division, awareness of place value (without this being explicitly mentioned) and, in the work moving between columns (Figure 11b), the beginnings of the ‘unitary method’ for solving problems. We make no claims about what students do or do not understand about multiplication and division. What we observe is that they have become energised by gaining fluency in symbolic manipulations and have developed awarenesses linked to these symbols. The partial understandings these students are working with did not appear to provoke anxiety and some of the joy and creativity of the exploration with these symbols can perhaps be guessed from their written work.

‘One possible challenge arising from this project, is to find other structured images and materials that can allow mathematical symbolism to be linked to relationships that can be perceived’

Conclusion

It seems clear from Case Study 3 that all students in the class were able to make the transition between a visual representation of a movement/relationship and its symbolic or formal description and go on to use this symbolism with confidence. The Gattegno chart was displayed throughout the project and some students asked for smaller, personal copies in order to support their work. The students were drawing on the image to aid their (symbolic) writing of their journeys. It took less than one hour for students to begin making mathematical connections between numbers and operations. There are some quite general lessons we feel able to draw, from experiences such as those documented in Case Study 3. The Gattegno chart offers a structuring of the number system that allows symbols (for multiplication and division) to be linked to relationships between rows and columns. Students gained symbolic fluency with these operators, unencumbered by the need for a laboured one-to-one linking to concrete objects. Students made connections in their written work between and within the formal representations themselves; this work, for some, was their first introduction to using multiplication and division symbols in school.

One possible challenge arising from this project, is to find other structured images and materials that can allow mathematical symbolism to be linked to relationships that can be perceived. Much has been written about the ambiguity of mathematical objects and the need for students to appreciate, for example, how symbols can stand for both processes and objects. There was seemingly no difficulty for students in the number ‘10’, for example, representing a position on the chart and being linked to an operation between positions on the chart. The presentation of symbols as linked to relationships perhaps offers students powerful access to their ambiguity.

As one teacher put it, on this project we have been “going down the language route” in learning mathematics. The Gattegno chart, for example, offers students access to the linguistic patterns in the number system that may be hidden when the focus is on the numbers 1 to 20. There is little difficulty in students working with hundreds once they have mastered the numbers 1 to 9 and there is energy and excitement from doing so.

We have seen consistently classrooms where all students have got excited about mathematics. It appears that when students experience some sense of control over the subject, for example, being able to choose their own starting point or line of enquiry then their enthusiasm explodes.

Adopting new practices is a challenge in any walk of life. All teachers began exploring new ideas in their classrooms by taking one lesson a week and devoting it to ‘Discovery Maths’ or ‘Becoming a Mathematician’. A challenge for teachers that was expressed and talked about on many occasions during the project was the possible tension between a focus on creative processes and the need for exam preparation. The Case Studies suggest these two agenda need not be in tension. One thing has been clear from our work over the last three years, the classrooms where the most radical change has taken place, in terms of a permanent focus on creative processes, have been the classrooms where achievement has been highest.

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5x5x5=creativity

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